Generic Programming: what, why and how

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What kind of generic?

In many languages, the function below is generic:

\[
\text{length} :: [a] \rightarrow \text{Int} \\
\text{length} [] = 0 \\
\text{length} (\_ : t) = 1 + \text{length} t
\]

In Haskell, however, we call \textit{length} a \textbf{polymorphic} function, and reserve the term \textit{generic} for something else...
Imagine you are writing software for helping students turn logic expressions into disjunctive normal form.
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You need:

- A description of the logic domain
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You need:

- A description of the logic domain
- Functionality on that domain:
  - Parsing and pretty-printing
  - Equality and top-level equality
  - Folding
  - Exercise generation
  - ...
Let’s get started, then:

```
data Logic = Logic → Logic  -- implication
    | Logic ↔ Logic  -- equivalence
    | Logic ∧ Logic  -- conjunction (and)
    | Logic ∨ Logic  -- disjunction (or)
    | Not Logic      -- negation (not)
    | Var String     -- variables
    | T              -- true
    | F              -- false
```
Exercise assistants: Logic III

\[\text{showLogic} :: \text{Logic} \rightarrow \text{String}\]
\[\text{showLogic} = \ldots\]
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\[
\text{showLogic} :: \text{Logic} \rightarrow \text{String} \\
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\]

\[
\text{parseLogic} :: \text{String} \rightarrow \text{Logic} \\
\text{parseLogic} = \ldots
\]
Exercise assistants: Logic III

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\text{showLogic} :: \text{Logic} \rightarrow \text{String} \\
\text{showLogic} = \ldots \\
\text{parseLogic} :: \text{String} \rightarrow \text{Logic} \\
\text{parseLogic} = \ldots \\
\text{type LogicAlgebra} a = \ldots \\
\text{foldLogic} :: \text{LogicAlgebra} a \rightarrow \text{Logic} \rightarrow a \\
\text{foldLogic} = \ldots \\
\text{evalLogic} :: (\text{String} \rightarrow \text{Bool}) \rightarrow \text{Logic} \rightarrow \text{Bool} \\
\text{evalLogic env l} = \text{foldLogic} \ldots l
\]
Exercise assistants: Logic III

\[ \text{showLogic :: Logic} \rightarrow \text{String} \]
\[ \text{showLogic} = \ldots \]

\[ \text{parseLogic :: String} \rightarrow \text{Logic} \]
\[ \text{parseLogic} = \ldots \]

\[ \textbf{type} \text{ LogicAlgebra} a = \ldots \]

\[ \text{foldLogic :: LogicAlgebra} a \rightarrow \text{Logic} \rightarrow a \]
\[ \text{foldLogic} = \ldots \]

\[ \text{evalLogic :: (String} \rightarrow \text{Bool)} \rightarrow \text{Logic} \rightarrow \text{Bool} \]
\[ \text{evalLogic env l} = \text{foldLogic} \ldots l \]

\[ \textbf{instance} \text{ Arbitrary Logic where} \]
\[ \text{arbitrary} = \ldots \]

\[ \ldots \]
Exercise assistants: Linear expressions I

Great! Your exercise assistant was a success and now you are asked to develop a tool to help students solving linear equations.

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▶ A description of the linear expressions domain
Exercise assistants: Linear expressions I

Great! Your exercise assistant was a success and now you are asked to develop a tool to help students solving linear equations.

You need:

- A description of the linear expressions domain
- Functionality on that domain:
  - Parsing and pretty-printing
  - Equality and top-level equality
  - Folding
  - Exercise generation
  - …
Let’s get started, then:

```haskell
data Expr = Con Rational    -- Constants
          | EVar String     -- Variables
          | Expr :+: Expr   -- Addition
          | Expr :-: Expr   -- Subtraction
          | Expr :*: Expr   -- Multiplication
          | Expr :/: Expr   -- Division
```
Exercise assistants: Linear expressions III

\[ \text{showExpr} :: \text{Expr} \rightarrow \text{String} \]
\[ \text{showExpr} = \ldots \]
Exercise assistants: Linear expressions III

\[\text{showExpr} :: \text{Expr} \to \text{String} \]
\[\text{showExpr} = \ldots\]

\[\text{parseExpr} :: \text{String} \to \text{Expr} \]
\[\text{parseExpr} = \ldots\]
Exercise assistants: Linear expressions III

\begin{align*}
  & \text{showExpr :: Expr } \to \text{ String} \\
  & \text{showExpr} = \ldots \\
  & \text{parseExpr :: String } \to \text{ Expr} \\
  & \text{parseExpr} = \ldots \\
  & \text{type ExprAlgebra a} = \ldots \\
  & \text{foldExpr :: ExprAlgebra a } \to \text{ Expr } \to \text{ a} \\
  & \text{foldExpr} = \ldots \\
  & \text{evalExpr :: (String } \to \text{ Rational) } \to \text{ Expr } \to \text{ Rational} \\
  & \text{evalExpr env e} = \text{foldExpr } \ldots \ e
\end{align*}
Exercise assistants: Linear expressions III

\[\text{showExpr} :: \text{Expr} \to \text{String}\]
\[\text{showExpr} = \ldots\]

\[\text{parseExpr} :: \text{String} \to \text{Expr}\]
\[\text{parseExpr} = \ldots\]

\textbf{type} \ \textit{ExprAlgebra} \ a = \ldots

\[\text{foldExpr} :: \text{ExprAlgebra} \ a \to \text{Expr} \to a\]
\[\text{foldExpr} = \ldots\]

\[\text{evalExpr} :: (\text{String} \to \text{Rational}) \to \text{Expr} \to \text{Rational}\]
\[\text{evalExpr \ env \ e} = \text{foldExpr \ \ldots \ e}\]

\textbf{instance} \ \textit{Arbitrary} \ \textit{Expr} \ \textbf{where}\]
\[\text{arbitrary} = \ldots\]

\ldots
Oops. After all your tool should deal with polynomials too. You need to add exponentiation to your datatype:

```
data Expr = Con Rational |
            EVar String |
            Expr :+: Expr |
            Expr :-: Expr |
            Expr :*: Expr |
            Expr :/: Expr |
            Expr :^: Expr -- Exponentiation
```
Exercise assistants: Polynomials...

Oops. After all your tool should deal with polynomials too. You need to add exponentiation to your datatype:

```
data Expr = Con Rational  
  | EVar String  
  | Expr :+: Expr  
  | Expr :->: Expr  
  | Expr :*: Expr  
  | Expr :/: Expr  
  | Expr :^: Expr   -- Exponentiation
```
Oops. After all your tool should deal with polynomials too. You need to add exponentiation to your datatype:

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           | EVar String
           | Expr :+: Expr
           | Expr :敢: Expr
           | Expr :*: Expr
           | Expr :/: Expr
           | Expr :^: Expr -- Exponentiation
```

Of course, now you also need to change all your functions...
Going generic

... is there no easier way to do this? ...
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...is there no easier way to do this?...

Yes! The answer is **Generic Programming**. With it you can:

- Write functions that work on any datatype
- Write common functionality once and for all
- Change your datatypes without changing your functions
- Avoid errors from code duplication
- ...
What is necessary for generic programming?

The essential ingredient is a reflection mechanism. We have to be able to inspect values and their types at runtime. Additionally, we have to be able to represent many different values in a uniform way. If we can map all values into a small set of datatypes, we can then define functions on this small set and they will work for every datatype.
Ingredients for Generic Programming I

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Additionally, we have to be able to represent many different values in a uniform way. If we can map all values into a small set of a datatypes, we can then define functions on this small set and they will work for every datatype.
Haskell’s **data** construct combines several features: type abstraction, type recursion, (labeled) sums, and (possibly labeled) products, but they are essentially **sums of products**.
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We can represent them using the following data types:

```
data a :+: b = L a | R b
data a :*: b = a :*: b
data Unit = Unit
infixr 5 :+: 
infixr 6 :*: 
```
Structure Types

We can use these structure types to encode Haskell data types:

```haskell
data Tree = Leaf | Node Tree Int Tree

type RTree = Unit :+: Tree :*: Int :*: Tree

data List a = Nil | Cons a (List a)

type RList a = Unit :+: a :*: List a
```
Generic values

We encode the values in the same way:

\[
\begin{align*}
\text{tree} &:: \text{Tree} \\
\text{tree} &= \text{Leaf} \\
\text{rtree} &:: \text{RTree} \\
\text{rtree} &= \text{L Unit} \\
\text{list} &:: \text{List Int} \\
\text{list} &= \text{Cons 2 Nil} \\
\text{rlist} &:: \text{RList Int} \\
\text{rlist} &= \text{R (2 :×: Nil)}
\end{align*}
\]
Types and structure types are isomorphic

A type is isomorphic to its structural representation type. For example, for the list data type we have:

\[
\begin{align*}
\text{from}_{\text{List}} & :: \text{List} \ a \rightarrow \text{RList} \ a \\
\text{from}_{\text{List}} (\text{Nil}) & = \text{L Unit} \\
\text{from}_{\text{List}} (\text{Cons} \ a \ as) & = \text{R} \ (a : \times : as) \\
\text{to}_{\text{List}} & :: \text{RList} \ a \rightarrow \text{List} \ a \\
\text{to}_{\text{List}} (\text{L Unit}) & = \text{Nil} \\
\text{to}_{\text{List}} (\text{R} \ (a : \times : as)) & = \text{Cons} \ a \ as
\end{align*}
\]

All the necessary infrastructure (RList, from\text{List} and to\text{List}) can be generated automatically.
Generic functions

A generic function can now be defined by induction on the structure of types, by writing cases for binary sums, binary products, nullary products, and primitives.

We use a GADT to unify the representation types into a single Rep:

```haskell
data Rep t where
  RSum :: Rep a → Rep b → Rep (a :+: b)
  RProd :: Rep a → Rep b → Rep (a :×: b)
  RUnit :: Rep Unit
  RInt   :: Rep Int
  RChar  :: Rep Char
```
Now we can define, say, generic equality:

\[ eq :: \text{Rep} \ a \rightarrow a \rightarrow a \rightarrow \text{Bool} \]

\[ eq \ (\text{RInt} \ i \ j) = eq_{\text{Int} \ i \ j} \]
\[ eq \ (\text{RChar} \ c \ d) = eq_{\text{Char} \ c \ d} \]
\[ eq \ (\text{RUnit} \ \text{Unit} \ \text{Unit}) = \text{True} \]
\[ eq \ (\text{RSum} \ r_a \ r_b) \ (\text{L} \ a_1) \ (\text{L} \ a_2) = eq \ r_a \ a_1 \ a_2 \]
\[ eq \ (\text{RSum} \ r_a \ r_b) \ (\text{R} \ b_1) \ (\text{R} \ b_2) = eq \ r_b \ b_1 \ b_2 \]
\[ eq \ (\text{RSum} \ r_a \ r_b) \_ \_ = \text{False} \]
\[ eq \ (\text{RProd} \ r_a \ r_b) \ (a_1 :\times: b_1) \ (a_2 :\times: b_2) = \text{eq} \ r_a \ a_1 \ a_2 \]
\[ \land \eq \ r_b \ b_1 \ b_2 \]
Generic equality II

But we are still lacking a case for arbitrary datatypes. When two types are isomorphic, the corresponding isomorphisms can be stored as a pair of functions converting back and forth—an embedding-projection pair:

\[
\text{data } \text{EP } d \ r = \text{EP} \ \{\text{from } :: (d \rightarrow r), \text{to } :: (r \rightarrow d)\}\]

But we are still lacking a case for arbitrary datatypes. When two types are isomorphic, the corresponding isomorphisms can be stored as a pair of functions converting back and forth—an embedding-projection pair:

\[
\textbf{data } EP \; d \; r = EP \{ from :: (d \rightarrow r), to :: (r \rightarrow d) \}
\]

We extend our representation type with a case for arbitrary types:

\[
\textbf{data } Rep \; t \; \textbf{ where } \\
\text{ ... } \\
RType :: EP \; d \; r \rightarrow Rep \; r \rightarrow Rep \; d
\]
Generic equality III

And add this case to the generic equality function:

\[ eq :: \text{Rep} \ a \to \text{a} \to \text{a} \to \text{Bool} \]
\[
\ldots
\]
\[
eq (\text{RType} \, \text{ep} \, r_a)\ t1\ t2 = \text{eq} \, r_a \, (\text{from ep} \, t1) \, (\text{from ep} \, t2)
\]
Generic equality III

And add this case to the generic equality function:

\[
\text{eq} :: \text{Rep } a \to a \to a \to \text{Bool}
\]

\[
\cdots
\]

\[
\text{eq } (\text{RType } ep \; r_a) \; t1 \; t2 = \text{eq } r_a \; (\text{from } ep \; t1) \; (\text{from } ep \; t2)
\]

As an example, for lists we have:

\[
\text{rList} :: \text{Rep } a \to \text{Rep } (\text{List } a)
\]

\[
\text{rList } r_a = \text{RType } (\text{EP from}_{\text{List}} \; \text{to}_{\text{List}})
\]

\[
(\text{RSum } \text{RUnit } (\text{RProd } r_a \; (\text{rList } r_a)))
\]
The basic principle here described can be explored in several different ways. We have seen a variant of Lightweight Implementation of Generics and Dynamics (LIGD). There are several other libraries for generic programming:

- Scrap Your Boilerplate (SYB)
- Uniplate
- Generics for the Masses (EMGM)
- Regular
- MultiRec
- ... and at least 7 others

These libraries vary in expressiveness, ease of use and understanding, and underlying mechanisms used.
Conclusions I

- Generic programming provides a way of reducing “boilerplate” code
- Functions are defined on the structure of datatypes and therefore work for every datatype
- If a datatype changes, the generic functions do not need to be adapted

A lot of work has been done in generic programming, and many functions are already available “for free”, such as generation of test data, (basic) parsing and pretty-printing, rewriting, etc.
Conclusions II

Current work at Utrecht University focuses on:

▶ Development of a powerful, easy to use and expressive generic programming library
▶ Applying generic programming to a large, showcase application
▶ Comparing performance of different approaches and investigating techniques for optimization of generic programs